

The Bridge to A level

Diagnosis

Worked

Solutions



1 Solving quadratic equations

Question 1

Solve $x^2 + 6x + 8 = 0$

$(x + 2)(x + 4) = 0$

$x = -2$ or -4

(2)

Question 2

Solve the equation $y^2 - 7y + 12 = 0$

Hence solve the equation $x^4 - 7x^2 + 12 = 0$

$$\begin{aligned}
 y^2 - 7y + 12 &= 0 \\
 (y - 3)(y - 4) &= 0 \rightarrow \underline{y = 3} \text{ or } \underline{y = 4} \\
 x^4 - 7x^2 + 12 &= 0 \rightarrow \text{Let } x^2 = y \\
 (x^2)^2 - 7x^2 + 12 &= 0 \rightarrow y^2 - 7y + 12 = 0 \rightarrow y = 3 \text{ or } y = 4 \\
 &\rightarrow x^2 = 3 \text{ or } x^2 = 4 \\
 &\rightarrow \underline{x = \pm\sqrt{3}} \text{ or } \underline{x = \pm 2}
 \end{aligned}$$

(4)

Question 3

(i) Express $x^2 - 6x + 2$ in the form $(x-a)^2 - b$

$$\begin{aligned}
 x^2 - 6x + 2 &= (x - 3)^2 - 9 + 2 \\
 &= \underline{(x - 3)^2 - 7}
 \end{aligned}$$

(3)

(ii) State the coordinates of the minimum value on the graph of $y = x^2 - 6x + 2$

Minimum point of $x^2 - 6x + 2$ is therefore $(3, -7)$

(1)

Total / 10



2 Changing the subject

Question 1

Make v the subject of the formula $E = \frac{1}{2}mv^2$

$$E = \frac{1}{2}mv^2$$

$$\Rightarrow 2E = mv^2$$

$$\Rightarrow \frac{2E}{m} = v^2$$

$$\pm \sqrt{\frac{2E}{m}} = v$$

(3)

Question 2

Make r the subject of the formula $V = \frac{4}{3}\pi r^3$

$$V = \frac{4}{3}\pi r^3$$

Get rid of fractions

$$3V = 4\pi r^3$$

make r^3 the subject.

$$\frac{3V}{4\pi} = r^3$$

cube both sides

$$\sqrt[3]{\frac{3V}{4\pi}} = r$$

(3)

Question 3

Make c the subject of the formula $P = \frac{c}{c+4}$

$$P = \frac{c}{c+4}$$

Get rid of fractions

$$\Rightarrow P(c+4) = c$$

Expand brackets

$$\Rightarrow Pc + 4P = c$$

Get terms with c on L.H.S., other terms on R.H.S.

$$Pc + 4P - c = 0$$

$$Pc - c = -4P$$

Factorise.

$$c(p-1) = -4P$$

Divide

$$c = \frac{-4P}{p-1} \quad \left(= \frac{4P}{1-p} \right)$$

(4)

Total / 10



3 Simultaneous equations

Question 1

Find the coordinates of the point of intersection of the lines $y = 3x + 1$ and $x + 3y = 6$

$$y = 3x + 1 \text{ and } x + 3y = 6$$

$$x + 3(3x + 1) = 6$$

$$x + 9x + 3 = 6$$

$$10x = 3$$

$$x = \frac{3}{10}$$

$$y = 3\left(\frac{3}{10}\right) + 1$$

$$= \frac{9}{10} + 1$$

$$= 1\frac{9}{10}$$

$(\frac{3}{10}, 1\frac{9}{10})$ or $(0.3, 1.9)$

(3)

Question 2

Find the coordinates of the point of intersection of the lines $5x + 2y = 20$ and $y = 5 - x$

Given $5x + 2y = 20$ & $y = 5 - x$.

Solving simultaneously

$$5x + 2(5 - x) = 20$$

$\Rightarrow 5x + 10 - 2x = 20$

$$\Rightarrow 3x = 10$$

$\Rightarrow x = \frac{10}{3}$, $y = 5 - \frac{10}{3} = \frac{5}{3}$

Note - if you round these fractions to decimals (10.3, 1.7) you lose a mark.

Pt. of intersection is $(\frac{10}{3}, \frac{5}{3})$

(3)

Question 3

Solve the simultaneous equations

$$x^2 + y^2 = 5$$

$$y = 3x + 1$$

Sub in $y = 3x + 1$ into equation 2.

$$x^2 + (3x + 1)^2 = 5$$

$$x^2 + (3x + 1)(3x + 1) = 5$$

$$x^2 + 9x^2 + 3x + 3x + 1 = 5$$

$$10x^2 + 6x + 1 = 5$$

$$10x^2 + 6x - 4 = 0$$

$(\div 2)$

$$5x^2 + 3x - 2 = 0$$

$$(5x - 2)(x + 1) = 0$$

$$x = \frac{2}{5} \text{ or } x = -1$$

When $x = \frac{2}{5}$

$$y = (3 \times \frac{2}{5}) + 1$$

$$= \frac{6}{5} + \frac{5}{5} = \frac{11}{5}$$

When $x = -1$

$$y = (3 \times -1) + 1$$

$$= -3 + 1$$

$$= -2$$

(4)

Total / 10



4 Surds

Question 1

(i) Simplify $(3 + \sqrt{2})(3 - \sqrt{2})$

$$\begin{aligned} & (3 + \sqrt{2})(3 - \sqrt{2}) \\ &= 3^2 + 3\sqrt{2} - 3\sqrt{2} - (\sqrt{2})^2 \\ &= 9 - 2 \\ &= 7 \end{aligned}$$

(2)

(ii) Express $\frac{1+\sqrt{2}}{3-\sqrt{2}}$ in the form $a + b\sqrt{2}$ where a and b are rational

$$\begin{aligned} \frac{(1 + \sqrt{2})}{(3 - \sqrt{2})} &= \frac{(1 + \sqrt{2})(3 + \sqrt{2})}{(3 - \sqrt{2})(3 + \sqrt{2})} \\ &= \frac{3 + \sqrt{2} + 3\sqrt{2} + (\sqrt{2})^2}{7} \\ &= \frac{3 + 4\sqrt{2} + 2}{7} \\ &= \frac{5}{7} + \frac{4}{7}\sqrt{2} \end{aligned}$$

To rationalise a denominator of form $(x + \sqrt{y})$ multiply top + bottom by $(x - \sqrt{y})$

(3)

Question 2

(i) Simplify $5\sqrt{8} + 4\sqrt{50}$. Express your answer in the form $a\sqrt{b}$ where a and b are integers and b is as small as possible.

$$\begin{aligned} \text{(i)} \quad & 5\sqrt{8} + 4\sqrt{50} \\ &= 5\sqrt{4}\sqrt{2} + 4\sqrt{25}\sqrt{2} \\ &= 5 \times 2\sqrt{2} + 4 \times 5\sqrt{2} \\ &= 10\sqrt{2} + 20\sqrt{2} \\ &= 30\sqrt{2} \end{aligned}$$

Always look for square number factors

(2)

(ii) Express $\frac{\sqrt{3}}{6-\sqrt{3}}$ in the form $p + q\sqrt{3}$ where p and q are rational

$$\begin{aligned} \frac{\sqrt{3}}{6 - \sqrt{3}} &= \frac{\sqrt{3}}{6 - \sqrt{3}} \times \frac{(6 + \sqrt{3})}{(6 + \sqrt{3})} \\ &= \frac{\sqrt{3} \times 6 + \sqrt{3}\sqrt{3}}{6^2 - (\sqrt{3})^2} \\ &= \frac{6\sqrt{3} + 3}{36 - 3} \\ &= \frac{3 + 6\sqrt{3}}{33} \\ &= \frac{3}{33} + \frac{6}{33}\sqrt{3} \\ &= \frac{1}{11} + \frac{2}{11}\sqrt{3} \end{aligned}$$

(3)

Total / 10



5 Indices

Question 1

Simplify the following

- (i) a^0 (1)
- (ii) $a^6 \div a^{-2}$ (1)
- (iii) $(9a^6b^2)^{-0.5}$ (3)

$$\begin{aligned}
 \text{(i)} \quad & \underline{a^0 = 1} \\
 \text{(ii)} \quad & a^6 \div a^{-2} = a^{6 - (-2)} \\
 & = \underline{a^8} \\
 \text{(iii)} \quad & (9a^6b^2)^{-1/2} = (3^2a^6b^2)^{-1/2} \\
 & = \underline{3^{-1}a^{-3}b^{-1}} \quad \left(= \frac{1}{3a^3b} \right)
 \end{aligned}$$

Question 2

- (i) Find the value of $\left(\frac{1}{25}\right)^{-0.5}$ (2)
- (ii) Simplify $\frac{(2x^2y^3z)^5}{4y^2z}$ (3)

$$\begin{aligned}
 \text{(i)} \quad & \left(\frac{1}{25}\right)^{-1/2} = (25)^{1/2} = \sqrt{25} = \underline{\pm 5} \\
 \text{(ii)} \quad & \frac{(2x^2y^3z)^5}{4y^2z} = \frac{2^5 x^{10} y^{15} z^5}{2^2 y^2 z^1} \\
 & = 2^{5-2} x^{10} y^{15-2} z^{5-1} \\
 & = 2^3 x^{10} y^{13} z^4 = \underline{8x^{10}y^{13}z^4}
 \end{aligned}$$

Total / 10



6 Properties of Lines

Question 1

A (0,2), B (7,9) and C (6,10) are three points.

(i) Show that AB and BC are perpendicular

$$\text{Grad of AB} = \frac{9-2}{7-0} = 1$$

$$\text{Grad of BC} = \frac{10-9}{6-7} = -1$$

Product of gradients = $1 \times -1 = -1 \rightarrow$ AB and BC perpendicular

(ii) Find the length of AC

$$(6-0)^2 + (10-2)^2 = AC^2$$

$$AC = 10$$

(3)

(2)

Question 2

Find, in the form $y = mx + c$, the equation of the line passing through A (3,7) and B (5,-1).
Show that the midpoint of AB lies on the line $x + 2y = 10$

$$m = \frac{-1-7}{5-3} = \frac{-8}{2} = -4$$

$$y = -4x + c$$

Substitute in (3,7)

$$\Rightarrow 7 = -4 \times 3 + c$$

$$19 = c$$

$$\Rightarrow \underline{y = -4x + 19}$$

$$\text{Midpoint of AB} = \underline{(4, 3)}$$

Sub. in to $x + 2y = 10$ & show that equation is true.

$$\underline{4 + 2 \times 3 = 4 + 6 = 10}$$

TRUE.

(5)

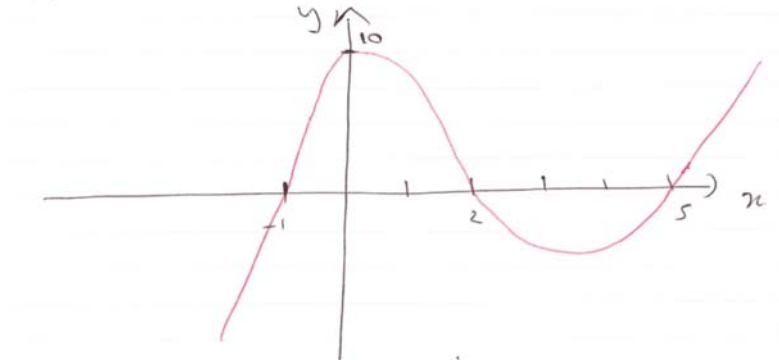
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7 Sketching curves

Question 1

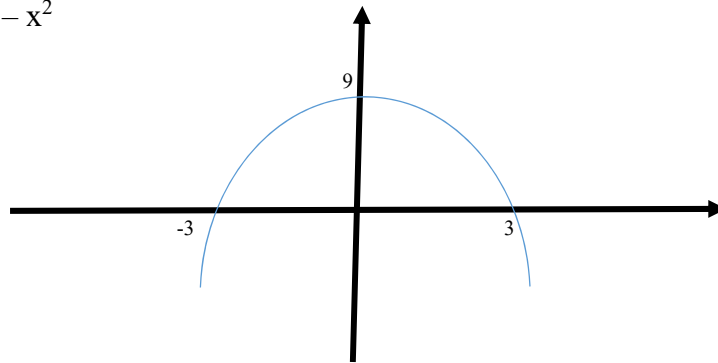
In the cubic polynomial $f(x)$, the coefficient of x^3 is 1. The roots of $f(x) = 0$ are -1, 2 and 5. Sketch the graph of $y = f(x)$



(3)

Question 2

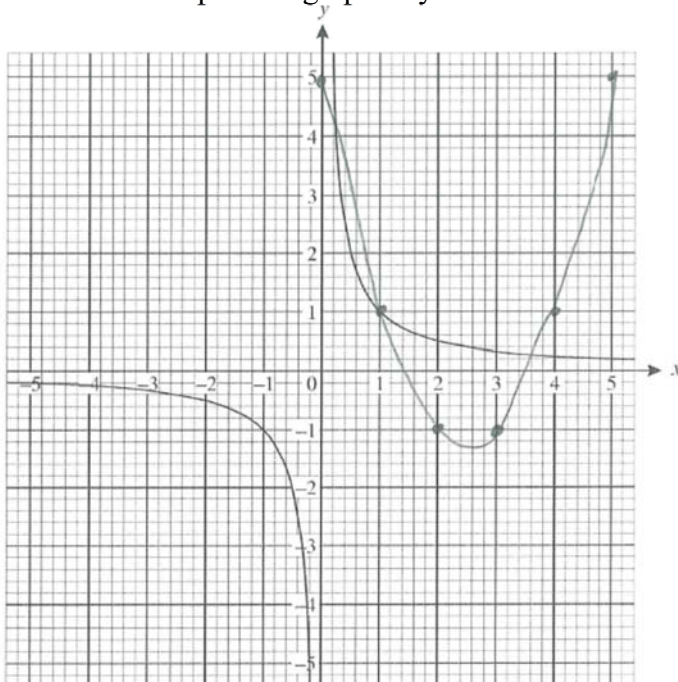
Sketch the graph of $y = 9 - x^2$



(3)

Question 3

The graph below shows the graph of $y = \frac{1}{x}$. On the same axes plot the graph of $y = x^2 - 5x + 5$ for $0 \leq x \leq 5$



x	0	1	2	3	4	5
x^2	0	1	4	9	16	25
$-5x$	0	-5	-10	-15	-20	-25
$+5$	$+5$	$+5$	$+5$	$+5$	$+5$	$+5$
y	5	1	-1	-1	1	5

(4)

Total / 10



8 Transformation of functions

Question 1

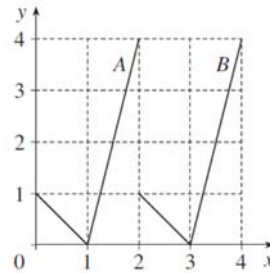
The curve $y = x^2 - 4$ is translated by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

Write down an equation for the translated curve. You need not simplify your answer.

$$y = (x-2)^2 - 4 \quad (2)$$

Question 2

This diagram shows graphs A and B.



- (i) State the transformation which maps graph A onto graph B

A movement of 2 to the right is
a translation of $\begin{pmatrix} +2 \\ 0 \end{pmatrix}$

- (ii) The equation of graph A is $y = f(x)$.

Which one of the following is the equation of graph B?

$$y = f(x) + 2$$

$$y = 2f(x)$$

$$y = f(x) - 2$$

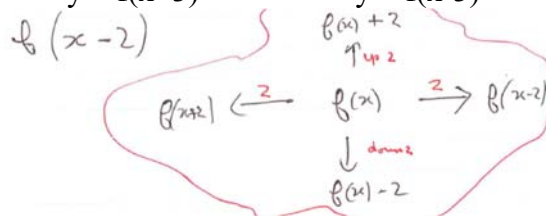
$$y = f(x+3)$$

$$y = f(x+2)$$

$$y = f(x-3)$$

$$y = f(x-2)$$

$$y = 3f(x)$$



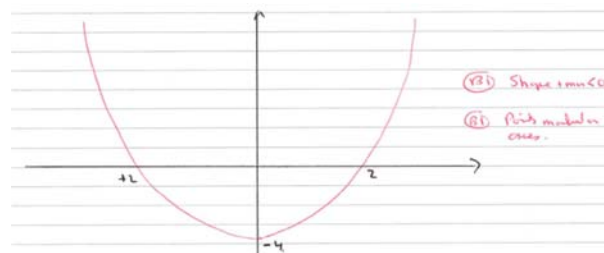
Answer $f(x-2)$ (2)

Question 3

- (i) Describe the transformation which maps the curve $y = x^2$ onto the curve $y = (x+4)^2$

• Translation (3) $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$ (or 4 units to the left) (3)

- (ii) Sketch the graph of $y = x^2 - 4$



(2)

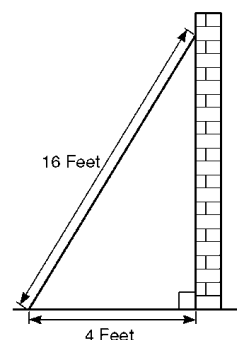
Total / 10



9 Trigonometric ratios

Question 1

Sidney places the foot of his ladder on horizontal ground and the top against a vertical wall. The ladder is 16 feet long.



The foot of the ladder is 4 feet from the base of the wall.

- (i) Work out how high up the wall the ladder reaches. Give your answer to 3 significant figures.

$$\sqrt{16^2 - 4^2}$$

$$\sqrt{256 - 16} \quad \text{correct substitution (M1)}$$

$$\sqrt{240}$$

$$15.49$$

$$15.5 \text{ (3sf)} \quad \text{(A1)}$$

(2)

- (ii) Work out the angle the base of the ladder makes with the ground. Give your answer to 3 sig fig

$$\cos x = \frac{4}{16} \quad \text{correct ratio and substitution (M1)}$$

$$\cos x = 0.25$$

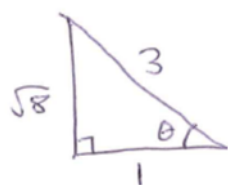
$$x = 75.522$$

$$x = 75.5^\circ \quad \text{(A1)}$$

(2)

Question 2

Given that $\cos \theta = \frac{1}{3}$ and θ is acute, find the exact value of $\tan \theta$

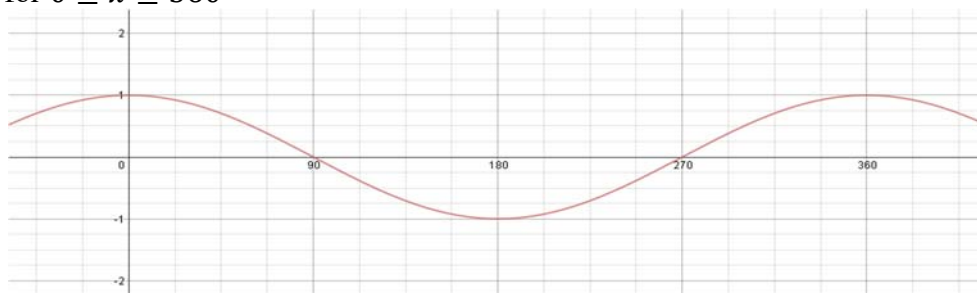


$$\tan \theta = \frac{\text{opp.}}{\text{Adj.}} = \frac{\sqrt{8}}{1} = \sqrt{8}$$

(3)

Question 3

Sketch the graph of $y = \cos x$ for $0 \leq x \leq 360^\circ$



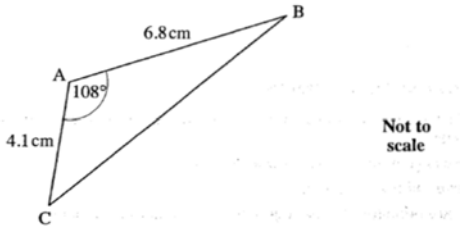
(3)

Total / 10



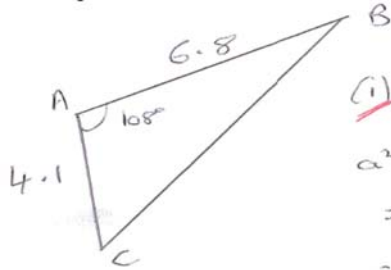
10 Sine / Cosine Rule

Question 1



For triangle ABC, calculate

(i) the length of BC



(i) By the Cosine Rule,

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$= 6.8^2 + 4.1^2 - 2 \times 6.8 \times 4.1 \times \cos 108$$

$$= 63.05 - - 17.23$$

$$= 80.28$$

$\Rightarrow a = \sqrt{80.28} = 8.960$

(3)

(ii) the area of triangle ABC

Area of a Triangle

$$= \frac{1}{2} ab \sin C$$

(Note: A diagram shows two sides and an included angle, with the text '2 sides + included angle' written next to it.)

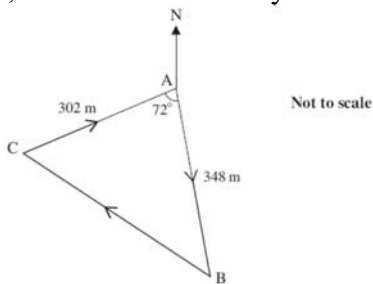
$$= \frac{1}{2} \times 4.1 \times 6.8 \times \sin 108$$

$$= 13.26$$

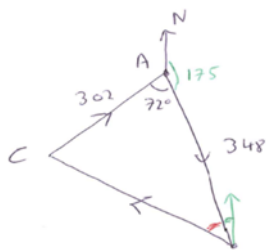
(3)

Question 2

The course for a yacht race is a triangle as shown in the diagram below. The yachts start at A, then travel to B, then to C and finally back to A.



Calculate the total length of the course for this race.



Use the Cosine Rule to find CB

$$CB^2 = 302^2 + 348^2 - 2 \times 302 \times 348 \times \cos 72$$

B. $CB = 384$

Total length = 384 + 650 = 1034m

(4)

Total / 10

